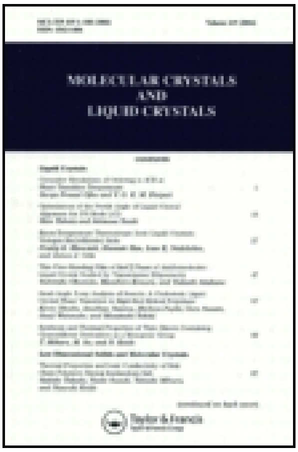

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The In-Plane Switching in the Nematic Cell

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The In-Plane Switching in the Nematic Cell

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We have analyzed the nematic liquid crystal reorientation in IPS mode and we present the result of theoretical study of this problem. In calculations we used the non uniform electric field profile[1] for inter-digital electrode configuration.

Keyword: liquid crystals; in-plane switching mode

INTRODUCTION

Liquid crystals as a media with the anisotropic properties (birefringence, dielectric and elastic properties) are now widely studied for their promising possibility in display applications. One serious problem of these displays lies in the limited viewing-angle characteristics. In particular, it was too difficult to achieve a symmetric high contrast ratio with no gray scale reversal as a function of viewing angles, because the director profile is significantly asymmetrical with respect to the substrate normal when an electric field is applied. The concept of liquid crystal displays employing IPS mode i.e., the electro-optical effect with interdigital electrodes at the lower substrate was developed in the 1970's. In 1990's this concept attracted scientific attention again[2-3]. This novel technology provides extremely wide viewing-angle characteristics[2-6].

In the IPS mode, an electric field is applied to the liquid crystals along the direction parallel to the plane of the substrates. The liquid crystals, aligned homogeneously between the substrates in the state without the electric field, are twisted by an in-plane electric field. The initially untwisted homogeneous configuration of the liquid crystals provides excellent light blocking. This behavior contributes to a high contrast ratio without gray scale reversal even for obliquely incident light at wide angles.

The threshold behavior and response characteristics of the liquid crystals in the IPS mode were studied[4-6] assuming uniform electric field and strong director anchoring at the cell boundaries. In this paper we analyze the behavior of nematic liquid crystals in the IPS mode taking into account the inhomogenity of applied electric field.

The structure of the paper is as follows. In the second section we employed our previous result[1] for two-electrode model to simplify the general problem and find the approximate electric field profile for in-plane electrode configuration using Fourier expansion. In the third section we calculated threshold voltage with averaged electric field obtained in the second section for nematic liquid crystal cell.

ELECTRIC FIELD PROFILE FOR IN-PLANE ELECTRODE CONFIGURATION

Simple model and boundary conditions

As was shown in the two-electrode model[1], the exact electric field distribution is not too inhomogeneous in between electrodes. This fact gives us the possibility to found the approximate electric field for the comb-shape electrodes formed on one lower substrate[3]. These electrodes produce inhomogeneous electric field nonparallel to the substrate, namely an in-plane electric field (Figures 1 - 2). Electric field potential Φ satisfies the Laplace equation $\Delta\Phi=0$ with the boundary conditions (Figure 2):

$$\Phi(y,z=0)=\Phi_0,\ \text{ if}$$
$$\frac{d_1}{2}+d_2+(2n-1)(d_1+d_2)\le y\le\frac{d_1}{2}+2n(d_1+d_2);$$
$$\Phi(y,z=0)=-\Phi_0,\ \text{ if}$$
$$\frac{d_1}{2}+d_2+2n(d_1+d_2)\le y\le\frac{d_1}{2}+(2n+1)(d_1+d_2);$$
$$n=0,\pm 1,\ldots \tag{1}$$

Φ_0 – is the electrodes potential.

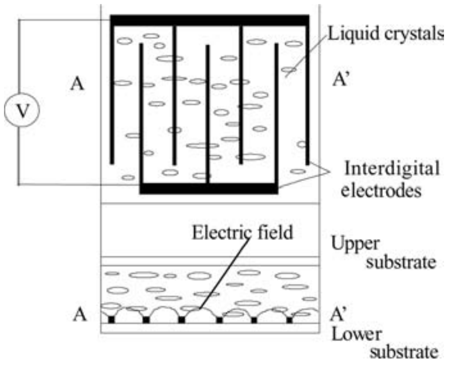


FIGURE 1. Schematic structure of liquid crystal cell for in-plane switching mode.

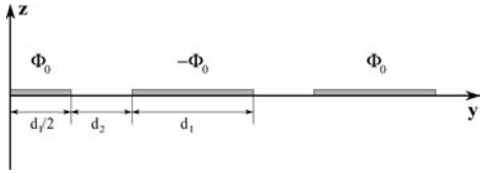


FIGURE 2. Interdigital electrodes and potential for the in-plane switching mode.

This problem was solved by separating the variables $\Phi(y,z) = Y(y) \times Z(z)$ and we got the following Fourier expansion for potential:

$$\Phi(u,v,\delta) = \sum_n C(n,\delta)\cos\left(\frac{n\pi u}{1+\delta}\right)\exp\left(-\frac{n\pi v}{1+\delta}\right); \qquad (2)$$

here *C (n, δ)* is the Fourier factor; $\delta = \frac{d_1}{d_2}$; $u = \frac{y}{d_2}$; $v = \frac{z}{d_2}$, and d_1 is the electrode width and d_2 is the electrode gap.

In the next step, we have to find the Fourier factor *C (n, δ)***.** This problem is too difficult to be solved exactly because the electric field distribution in between electrodes space at $z = 0$ (the lower substrate):

$$\frac{d_1}{2} + n(d_1 + d_2) \le y \le \frac{d_2}{2} + \left(n + \frac{1}{2}\right)(d_1 + d_2); \quad n = 0, \pm 1, ..$$

is unknown.

To simplify the problem (1) we hypothesized that potential Φ *(y, z = 0)* has a linear dependence on *y* in between the electrode space (the reason for this assumption is the quasi-linear dependence in a two-electrode model[1]):

$$\Phi(y, z = 0) = \Phi_0\left(\frac{d_1 + d_2}{d_2} - \frac{2}{d_2} y\right),$$

Where $\frac{d_1}{2} + n(d_1 + d_2) \le y \le \frac{d_2}{2} + \left(n + \frac{1}{2}\right)(d_1 + d_2); \quad n = 0, \pm 1, ..$

Finally we get the following solution for the above boundary value problem:

$$\Phi(u, v, \delta) = \Phi_0 \sum_m A(m, \delta) \cos\left(\frac{m\pi u}{1 + \delta}\right) \exp\left(-\frac{m\pi v}{1 + \delta}\right); \tag{3}$$

where *A (m,* δ*)* is the new Fourier factor for the electric field potential Φ distribution at the lower substrate.

The condition that the potential given by (2) and (3) has to be the same at the lower substrate, results in the following expression for Fourier factor *A (m,* δ*):*

$$A(m, \delta) = \frac{4}{m\pi} \sin\left(\frac{m\pi}{2}\right)\left[2\frac{1 + \delta}{m\pi} \sin\frac{m\pi}{2(1 + \delta)} - \cos\frac{m\pi}{2(1 + \delta)}\right] \tag{4}$$

The total number of *A (m,* δ*)* (4) in the full solution is infinite, in approximate solution we can truncate the series (however the price of this truncation is the oscillation in the Figures 3 and 4). In Figure 3 we show the dimensionless potential Φ/Φ_0 as the function of *u* and *v* at $d_1/d_2=0.5$ and in Figure 4 we plot the dimensionless electric field components $E_y d_2/\Phi_0$ and $E_z d_2/\Phi_0$ as the function of *u* and *v* at $d_1/d_2=0.5$

Respectively for electric field $E = -\nabla\Phi$ we got for $z \ge 0$:

$$E_y(u,v,\delta)=\frac{1}{d_2}\Phi_0\sum_n A(n,\delta)\frac{n\pi}{1+\delta}\sin\frac{n\pi u}{1+\delta}\exp\left(-\frac{n\pi v}{1+\delta}\right);$$
$$E_z(u,v,\delta)=\frac{1}{d_2}\Phi_0\sum_n A(n,\delta)\frac{n\pi}{1+\delta}\cos\frac{n\pi u}{1+\delta}\exp\left(-\frac{n\pi v}{1+\delta}\right);$$

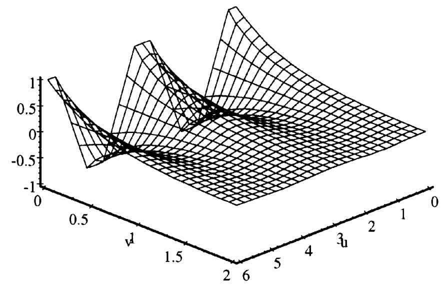
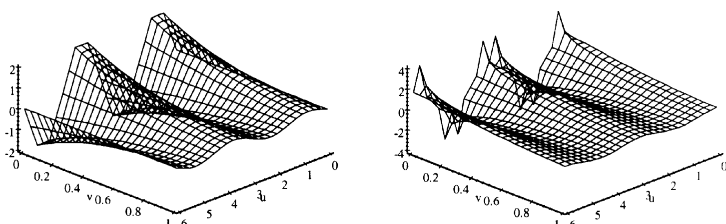


FIGURE 3. Dimensionless potential Φ/Φ_0 as the function of u and v at $d_1/d_2=0.5$



a) b)

FIGURE 4. Dimensionless electric field components $E_y d_2/\Phi_0$ and $E_z d_2/\Phi_0$ as the function of u and v at $d_1/d_2=0.5$

The averaging of electric field

Below we shall need an average over the cell volume values of the electric field $\boldsymbol{E}$:

$$\left\langle E_y(\delta,\gamma)\right\rangle=\frac{1}{\gamma}\frac{2}{1+\delta}\int_0^{\delta+1}du\int_0^{\gamma}E_y(u,v,\delta)dv=$$
$$=\frac{2\Phi_0}{\gamma}\sum_m A(m,\delta)\frac{2}{m\pi}\sin^2\frac{m\pi}{2}\left(1-\exp\left(-\frac{\gamma m\pi}{1+\delta}\right)\right), \qquad (5)$$

where $\gamma = \dfrac{H}{d_2}$ and H is the cell thickness.

For $<E_z(\delta, \gamma)>$ the domain of averaging is slightly different, because $E_z(u, v, \delta)$ periodically changes its sign, but for liquid crystal director the sign of $E_z(u, v, \delta)$ is irrelevant.

$$\langle E_z(\delta,\gamma)\rangle = \frac{1}{\gamma}\frac{2}{1+\delta}\int_0^{(\delta+1)/2} du \int_0^{\gamma} E_z(u,v,\delta)dv =$$
$$= \frac{2\Phi_0}{\gamma}\sum_m A(m,\delta)\frac{1}{m\pi}\sin\frac{m\pi}{2}\left(1-\exp\left(-\frac{\gamma m\pi}{1+\delta}\right)\right).$$

Figure 5 a, b shows the relation $\dfrac{\langle E_y(\delta,\gamma)\rangle}{\langle E_z(\delta,\gamma)\rangle}$ as the function of γ at $\delta = 1; 0.05$ respectively.

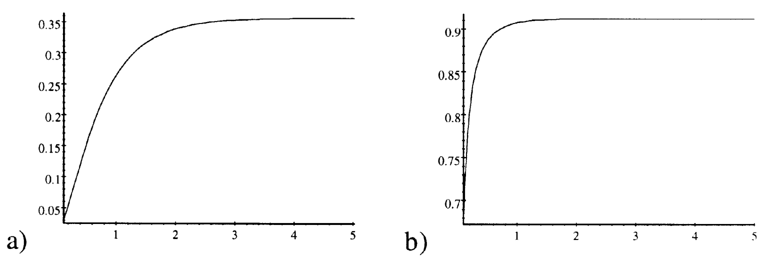


FIGURE 5. a) $\dfrac{\langle E_y(\delta,\gamma)\rangle}{\langle E_z(\delta,\gamma)\rangle}$ as the function of γ at $\delta = 1$ and b) $\delta = 0.05$.

THRESHOLD VOLTAGE

Total elastic free energy

Detailed switching behavior of liquid crystals following applied electric field in the IPS mode were analyzed by O-he[6] from the viewpoint of the Freedericksz transition, using the continuum elastic theory and supposing the electric field to be homogeneous and to have only E_y component. Figure 6 shows the real geometry of the director deformation by an in-plane electric field.

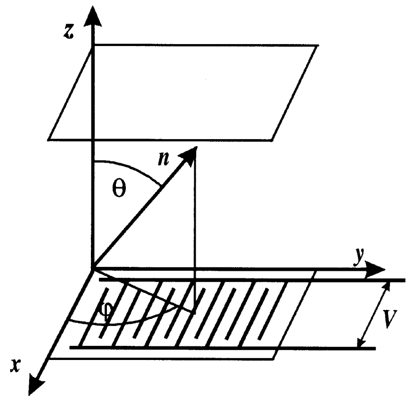


FIGURE 6. Geometry of the director deformation by an in-plane electric field.

In the switched off state the director is given by the unit vector $\boldsymbol{n} = (1, 0, 0)$. On applying the voltage director rotates and it is convenient to describe the director profile by two angles φ and θ.

$$n = (\sin\theta(y,z)\cos\varphi(y,z), \sin\theta(y,z)\sin\varphi(y,z), \cos\theta(y,z) \qquad (6)$$

The total elastic free energy F of the nematic liquid crystal in one elastic constant approximation and with strong director anchoring one can write[8]:

$$F = \frac{K}{2}\int\left[(\nabla\vec{n})^2 + (\vec{\nabla}\times\vec{n})^2\right]dV - \frac{\varepsilon_a}{8\pi}\int\left(n\vec{E}\right)^2 dV \,;$$

or in terms of φ and θ:

$$F = \frac{K}{2}\int_0^L\int_0^{d_1+d_2}\left\{\left[\left(\frac{\partial\theta}{\partial y}\right)^2 + \left(\frac{\partial\theta}{\partial z}\right)^2 + \left(\left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right)\sin^2\theta\right]\right.$$
$$\left. + 2\sin^2\theta\cos\varphi\left(\frac{\partial\theta}{\partial y}\frac{\partial\varphi}{\partial z} - \frac{\partial\varphi}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\} -$$

$$-\frac{\varepsilon_a}{8\pi}\int_0^L\int_0^{d_1+d_2}\left(E_y^2\sin^2\theta\sin^2\varphi+E_z^2\cos^2\theta+2E_yE_z\sin 2\theta\sin\varphi\right)dydz$$

where K is the elastic constant, E_y, E_z are the electric field components, and ε_a denotes the anisotropy of the LC dielectric tensor.

Ostrogradsky's equations

To find the equilibrium director profile that realizes the minimum of total free energy we have to solve the following Ostrogradsky's equations[9]:

$$\frac{K}{2}\left(2\Delta\theta-\sin 2\theta\left(\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right)\right)+$$
$$\frac{\varepsilon_a}{8\pi}\left(\sin 2\theta\left(E_y^2\sin^2\varphi-E_z^2\right)+2E_yE_z\cos 2\theta\sin\varphi\right)=0\,,$$
$$\frac{K}{2}\left(2\sin 2\theta\left(\frac{\partial\theta}{\partial y}\frac{\partial\varphi}{\partial y}+\frac{\partial\theta}{\partial z}\frac{\partial\varphi}{\partial z}\right)+2\sin^2\theta\Delta\phi\right)+$$
$$\frac{\varepsilon_a}{8\pi}\left(E_y^2\sin^2\theta\sin 2\varphi+E_yE_z\sin 2\theta\cos\varphi\right)=0$$

with the boundary conditions:

$$\varphi\ (y, z=0)=\varphi\ (y, z=H)=0;$$
$$\varphi\ (y=0, z)=\varphi\ (y=d_1+d_2, z)=0;$$
$$\theta\ (y, z=0)=\theta\ (y, z=H)=\pi/2;$$
$$\theta\ (y=0, z)=\theta\ (y=d_1+d_2, z)=\pi/2.$$

Threshold voltage

To study the stability of the initial director profile we linearised those equations with respect to φ and $\psi=\frac{\pi}{2}-\theta$ and got the following system of separated equations for small angles ψ and φ:

$$K\Delta\psi+\frac{\varepsilon_a\psi}{4\pi}E_z^2=0;\qquad K\Delta\varphi+\frac{\varepsilon_a\varphi}{4\pi}E_y^2=0 \tag{7}$$

To proceed further we approximate the electric field components by their averaged value $\langle E_y \rangle$ and $\langle E_z \rangle$. Now from the last equation it is easy to find the averaged value of the critical field $\langle E_y \rangle_{th}$ at which the twist angle φ just begins to change:

$$\langle E_{y,th} \rangle^2 = \left(\frac{4\pi K}{\varepsilon_a} \right) \left(\frac{\pi^2}{H^2} + \frac{\pi^2}{(d_1 + d_2)^2} \right),$$

the threshold voltage is given then by (5)

$$\Phi_{0,th} = \langle E_{y,th} \rangle \frac{d_2}{g(\delta, \gamma)},$$

where $g(\delta,\gamma)$ is the ratio:

$$g(\delta,\gamma) = \frac{1}{\gamma} \sum_m A(m,\delta) \frac{2}{m\pi} \sin^2 \frac{m\pi}{2} \left(1 - \exp\left(-\frac{\gamma m\pi}{1+\delta} \right) \right).$$

The Oh-e's results[6] for threshold electric field and threshold voltage are: $E^{IPS} = \frac{\pi}{H} \sqrt{\frac{4\pi K}{\varepsilon_a}}$, $\Phi^{IPS} = E^{IPS} d_2$.

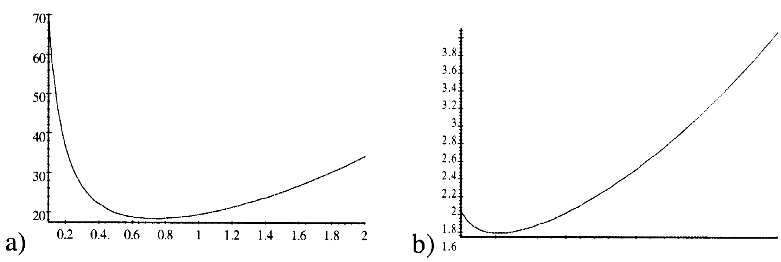


FIGURE 7. a) Φ_{th}/Φ^{IPS} at $\delta = d_1/d_2 = 1$ and b) at $\delta = d_1/d_2 = 0.1$

To compare the results of our calculations with Oh-e's one[4-6] we can write down the ratio between them:

$$\frac{\Phi_{th}}{\Phi^{IPS}} = \frac{1}{g(\delta,\gamma)}\sqrt{1+\frac{\gamma^2}{(1+\delta)^2}}\,.$$

Figure 7 a, b presents the ratio $\frac{\Phi_{th}}{\Phi^{IPS}}$ as the function of $\gamma = \frac{H}{d_2}$ at $\delta = \frac{d_1}{d_2}$ = *1; 0.1.*

CONCLUSIONS

Threshold voltage as it is seen from the Figure 7 may be significantly different from that predicted by simple theory of Oh-e[2-6], which doesn't care about the electric field inhomogeneity in the liquid crystal cell.

In figure 8 we plot the ratio $\frac{\Phi_{th}}{\Phi_{Oh-e}}$ that depend on $\delta = \frac{d_1}{d_2}$ and $\gamma = \frac{H}{d_2}$.

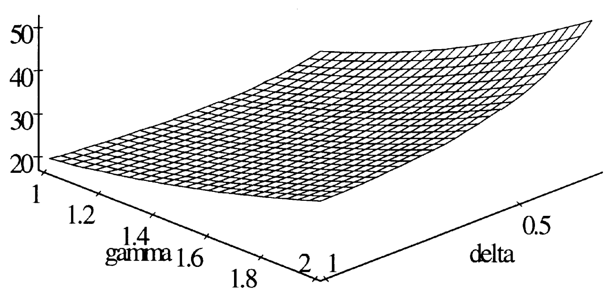


FIGURE 8. The ratio $\frac{\Phi_{th}}{\Phi_{Oh-e}}$ depend on $\delta = \frac{d_1}{d_2}$ and $\gamma = \frac{H}{d_2}$.

There exists optimal set of ratios δ, γ that corresponds to minimal value of the threshold voltage. It is better to have a higher value of δ, and a smaller value of γ. This last result is easily understandable. Smaller γ corresponds to more homogeneous E_y within cell thickness because of the exponential decay of the electric field components with the distance from the electrodes. At higher values of δ we have a smaller electrode gap and therefore higher electric field in between them.

Our results on operating voltage dependence on the cell thickness and electrode gap agrees with the experimental data of Matsumoto et al.[3].

Acknowledgments
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